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Coordination numbers of alkali metal ions in aqueous solutions

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Abstract

The level of complexity with which any biological ion interaction mechanism can be investigated, whether it is a binding mechanism in proteins or a permeation mechanism in ion channels, is invariably limited by the state-of-the-art of our understanding of the characteristic properties of ion solvation. Currently, our understanding of the energetic properties of ion solvation in aqueous phase is considered adequate enough to have helped us obtain satisfactory descriptions of the role of energetics in several biological ion interaction processes. In contrast, the lack of consensus among all the experimental structural hydration data determined more than 10 years ago, particularly regarding ion hydration numbers, has limited us to nothing better than speculation regarding the roles of local spatial environments in these mechanisms. Here we revisit experimental and theoretical work applied to probe hydration numbers of three alkali metal ions, Li^+, Na^+ and K^+, and analyze them to clarify the current state-of-the-art of our understanding of their structural hydration properties. We find that with substantial improvements over the past 10 years in areas of experimental techniques, data analysis strategies, and theoretical and computational approaches for interrogating ion hydration structures, there is now growing consensus regarding the hydration numbers of these ions. We see that under physiological conditions, *ab initio* methods suggest that all three ions prefer strong coordination with exactly 4 water molecules, a result we find consistent with some older experimental measurements. *Ab initio* molecular dynamics (AIMD) simulations invariably identify additional “loosely” coordinated water molecules at the far slopes of the principle maxima of the radial distribution profiles for Na^+ and K^+ ions. We suggest that these statistical admixtures of additional oxygen atoms have resulted in the most recent experimentally determined hydration numbers of Na^+ ions to be 5 and K^+ ions to be 6.

Keywords: Hydration numbers; Molecular dynamics; Quasi-chemical theory; Force fields; Ab initio; Alkali metal ions

1. Introduction

The characterization of ion solvation properties in aqueous phases has been an area of active research and interest for close to a century. Research in this area began as early as the 1930s since it was of particular interest to chemists who pursued investigations of chemical reactions in aqueous media. Their investigations required a detailed understanding of the nature of ion solvation in terms of both energetics as well as the formation of solvent microstructures around ions. To achieve this, comprehensive studies were carried out in water that included measurements of ion–water interaction energies, aqueous phase ion solvation energies, partial molar volumes of ions, average hydration numbers of ions, and ion–water distances. These studies now take on even greater significance as current biological research targets a molecular level understanding of ion solvation in biological environments, where interest lies in the mechanisms controlling ion binding to biomolecules and selective ion permeation through narrow channel proteins. A precise molecular level understanding of these biological processes holds tremendous potential for yielding spatial and electrical design parameters for next-generation biomimetic devices that control ion movements between aqueous and biological environments (for example, see Ref. [1]). These devices can in turn provide improved solutions to such diverse problems as water desalination and implantable electrical power generation.

Since water is the reference environment in biology, a precise molecular knowledge of ion hydration properties is a necessary prerequisite for developing mechanistic conclusions about

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biological control of ion movements. Although not strictly separable, many investigations have addressed ion hydration energetics apart from the geometries of their hydrated structures. In the past few decades, the energetic properties of ion hydration have been considered sufficiently understood so as to not hamper our abilities to describe the role of energetics in several physical and biological phenomena. They have also been appropriately parameterized into classical force fields and are now used on a regular basis to investigate diverse scientific and engineering problems. At the same time, however, a distinct lack of consensus [2] characterizes the structural properties of hydrated ions, especially their hydration numbers determined more than 10 years ago by experimental methods. Even the hydration properties of the simple alkali metal ions, though repeatedly investigated using multiple theoretical and experimental techniques, yet appear to reflect ambiguities in terms of hydration structure. This dissension limits our abilities to assign concrete and unambiguous roles for the structural effects of local environments on ion phenomena, a critical issue in understanding the molecular mechanisms driving ion binding and selective permeation in biological systems.

Here we summarize the older literature and consider current investigations of the hydration properties of three simple alkali metal ions: Li^+, Na^+, and K^+. With improvements in methods for probing liquid state structure, we find that the earlier confusion is giving way to a growing consensus, which may surprise some investigators, regarding the hydration numbers of these ions.

2. Experimental hydration numbers

Table 1 lists hydration numbers determined experimentally for the first three alkali metal ions more than 10 years ago. Hydration numbers reflect the average number of water molecules occupying the nearest coordination volume around an ion. Prior to 1996, we see from Table 1 that the experimental hydration numbers of Li^+ ions vary between the values of 3 and 6. The hydration numbers of Li^+ were considered to be dependent on salt concentrations during this period [3,4], with higher salt concentrations resulting in lower hydration numbers. Since these dependencies were measured well beyond salt concentrations of 1 M in all cases, they are not directly relevant to investigations of biological systems. Over the same time frame, scattered hydration numbers appear for Na^+ ions as well, with values all the way from 4 through 8. Note that the hydration number of Na^+ evaluated from neutron diffraction data stands out as exceptionally different from those determined using any other experimental technique. Hydration numbers for K^+ ions similarly span the range between 4 and 8, with the neutron diffraction data once again producing an exceptionally different value.

Both neutron and X-ray diffraction techniques determine [5] differential scattering cross-sections that are proportional to weighted sums of partial structure factors in polyatomic systems, like salt solutions. These partial structure factors can in theory be determined with high accuracy, and can yield accurate partial pair-distribution functions between ions (M) and water oxygen atoms (O), ($g_{\mathrm{M-O}}(r)$). Following the traditional statistical definition of hydration number, these partial pair-distribution functions (or radial distribution functions) can then be integrated up to their first absolute minima, which should define the nearest volume of coordinating solvent molecules, to obtain hydration numbers ($n_{\mathrm{M-O}}$) of ions. This entire process, however, requires very high spectral resolutions to define the critical first peaks and minima in radial distribution functions and thus to obtain accurate hydration numbers. Other challenges that further compound structural inferences arise in obtaining absolute normalizations of measured intensities and treating momentum truncations during data analysis. These difficulties make diffraction measurements on liquids particularly challenging and data analysis efforts generally complicated, requiring a variety of delicate corrections and careful interpretations to infer ion coordination numbers. Presumably these are the challenges that resulted in exceptional ambiguities seen in hydration data numbers estimated in the past [6].

In the last 10 years or so, several developments have significantly advanced these methods both in terms of data gathering and data analysis (see Ref. [6] for the most recent review). Table 2 summarizes the consequent experimental results obtained most recently for the hydration numbers of the small alkali metal ions: 4 for Li^+, 5 for Na^+, and 6 for K^+ ions. Note that the hydration number of 4 for Li^+ ion derives from using older neutron diffraction data [4], from which a hydration number of 6 had been inferred, and reanalyzing it using a more sophisticated data analysis technique [7]. In general, we note that advancements in

Table 1
Hydration numbers of ions estimated from experiments carried out prior to the year 1996

M^+	$n_{\mathrm{M^+O}}$	Method	Year [Ref.]
Li^+	4	Neutron diffraction	1973 [61]
	4	X-ray	1973 [62]
	4	X-ray	1975 [63]
	4	Raman spectroscopy	1978 [64]
	4	Neutron diffraction	1979 [65]
	4	X-ray	1980 [66]
	3–6 [a]	Neutron diffraction	1980 [3]
	6	X-ray	1981 [67]
	5	X-ray	1983 [68]
	4	X-ray	1983 [69]
	4	X-ray	1984 [70]
	6	X-ray	1987 [71]
	5	Neutron diffraction	1991 [72]
	3–6 [a]	Neutron diffraction	1996 [4]
Na^+	4	X-ray	1975 [73]
	4	Raman spectroscopy	1978 [64]
	6	X-ray	1980 [74]
	8	Neutron diffraction	1980 [75]
	4	X-ray	1980 [66]
	5	X-ray	1989 [76]
K^+	6	X-ray	1957 [77]
	4	X-ray	1958 [78]
	5	X-ray	1969 [79]
	6	X-ray	1980 [66]
	8	Neutron diffraction	1980 [75]

[a] Dependence of hydration numbers on salt concentrations: concentrations close to 1 M correspond to hydration numbers of 6, and higher concentrations correspond to lower hydration numbers.

Table 2
Hydration numbers of ions estimated from most recent experiments

M^+	n_{M^+O}	Method	Year [Ref.]
Li^+	4	Neutron diffraction [a]	2006 [7]
Na^+	5	Neutron diffraction	2006 [42]
K^+	6	Neutron diffraction	2001 [80]
	6	Neutron diffraction	2006 [81]

[a] Pairwise radial distribution functions re-evaluated using more sophisticated analysis methods [7] from older [4] neutron diffraction data.

neutron diffraction techniques and data analysis methods lead to coordination numbers considerably lower than those determined earlier using the same experimental methodology. Considering these hydration numbers to be the best experimental estimates to date, analysis of coordination numbers determined by diffraction reveals a unique pattern in ion hydration numbers. There appears to be a distinct structural feature associated with each hydrated ion wherein each unit move down the periodic table increases the hydration number by the same amount.

While experimental diffraction studies often rely on molecular models for support in data interpretation, computational and theoretical approaches can be used independently of experiments to probe the structural features of hydrated ions.

3. Theoretical approaches to probe hydration structures

3.1. Molecular dynamics employing parameterized force field models

It is a common misconception that a molecular dynamics calculation employing a parameterized (empirical) force field can be utilized to *predict* the structural properties of ion solvation. Use of such methods to probe the effect of parameters and functions in order to *understand* the nature of ion solvation is, however, an entirely different matter. In the traditional method of constructing force fields, which is by no means a straightforward task, the solvation properties deemed necessary for a certain investigation are plugged in via optimization of parameters belonging to a set of mathematical expressions chosen specifically to describe those solvation properties. Therefore, such force fields can at best be considered a reflection of our current understanding of the nature of particle interactions, although at the same time they are constrained by limitations in computing power. This implies that, in principle, such parameterized force fields should only be utilized to probe those properties of ion solvation for which they were originally parameterized, and should *not* be used for the purpose of predicting other solvation properties of ions.

For the sake of argument, we consider here two of the most commonly used ion force fields in theoretical biology, one that has been incorporated [8] into the CHARMM27 [9] package of force fields and the other that has been incorporated [10] into the OPLS-AA [11] package of force fields. In both these force fields, the overall strategy employed to generate ion parameters consisted of two separate steps. In the first step, the parameters describing water molecule interactions were optimized to reproduce certain experimental bulk water properties, and then in the next step these preset water parameters were used to optimize the 6–12 Lennard–Jones parameters of the ions. Obvious assumptions made in the construction of the ion force fields were that the 6–12 Lennard–Jones functional form and the previously optimized functions and parameters describing the water molecules could capture the new interaction of water molecules with ions. In adherence to tradition, in both cases the 6–12 Lennard–Jones parameters of Na^+ and K^+ ions were simply handpicked to reproduce a subset of ion hydration properties, the experimental values of hydration free energies and ionic radii, without any special care administered to reproduce other properties like hydration numbers.

Table 3 lists the hydration numbers of Na^+ and K^+ ions obtained [12] as a result of using these force fields. We clearly see that the simulated hydration numbers are higher than those estimated by the most recent experiments. Moreover, the computed K^+ ion hydration numbers of 7 were in fact never estimated by any of the previous experiments. Furthermore, due to the non-linear nature of Lennard–Jones interactions, the two experimental observables used as targets for parameterization could just as well have been reproduced by an alternate set of parameters, which implies that reproducing two experimental hydration properties does not guarantee that all other hydration properties would be simultaneously and automatically reproduced. Therefore these force fields are appropriately applied to simulate the energetic properties of ion solvation, but not to predict structural properties like radial distributions of ligand molecules around ions. When such limitations of using these force fields are overlooked, inconsistent results can arise. For example, a force field used to simulate the energetic solvation properties of a certain ion may in fact simultaneously simulate the structural properties of an entirely different ion. Thus interpretations regarding the structural properties of ion solvation derived from simulations employing such force fields should be dealt with carefully as they can be misleading. In the cases of Na^+ or K^+ ions, this is true even if that force field predicts a coordination number that is off by a single unit since the most recent experimental data show that these ions have characteristically distinct hydration numbers that differ by exactly one unit.

The two force fields considered above employ pair-additive potentials to describe interactions between ions and water molecules, which means the interaction between an ion and any given water molecule is assumed to be independent of all other atoms in the system. Several investigations [13–26], some of which were carried out as early as the 1970s, have identified certain non-pair-additive interactions that are critical to describing atomic-level details of ion solvation. These include the classical dipole polarizabilities of water molecules and the influence of

Table 3
Hydration numbers of ions computed [12] using two of the most commonly used empirical (parameterized) force fields in theoretical biology

M^+	CHARMM27 [8]	OPLS-AA [10]
Na^+	6	6, 7
K^+	7	7

Both these ion force fields describe ion water interactions using simple 6–12 Lennard–Jones functions, and do not explicitly account for atomic polarizabilities or multi-body effects on ion–water interactions.

other water molecules on the interaction between a given ion–water pair (many-body interactions). Inclusion of such effects can be expected to enhance the predictive capabilities of empirical force fields. Even with access to large computing resources, however, this is not a straightforward task since the degree of required theoretical accuracy must be balanced with computational efficiency. Inclusion of such effects generates complicated target potential surfaces that require the use of more sophisticated approaches for optimization purposes, and higher dimensionalities in mathematical functions that increase computational expense tremendously. This area of research is being actively pursued across several laboratories, and there is still much work needed to arrive at a single justifiable approach toward constructing such force fields [27–32].

Nonetheless, several noteworthy attempts have been made toward construction of such enhanced force fields that incorporate these additional interactions. Table 4 lists the hydration numbers of ions computed as a result of using these enhanced force fields. We see that in the case of Li^+ ions, the computed hydration numbers exactly match those estimated by recent experiments. For Na^+ and K^+ ions, however, the computed hydration numbers are still almost always higher than those obtained from the most recent experiments. This clearly exemplifies our earlier argument that work is still required to capture accurately the details of ion solvation when using empirical force fields. In such a scenario, the role of making predictive statements regarding structural details of ion solvation mechanisms is better left to either experiments or higher levels of theory that do not explicitly rely on parameterization of particle interactions. In the following section, we review the work done so far using *ab initio* methods to determine hydration numbers of the first three alkali metal ions.

3.2. Ab initio methods

Ab initio (quantum) calculations do not rely on parameterization to the extent or variety present in empirical force fields [33]. The nature of parameterizations present in the functionals used in density functional methods to describe correlated electron interactions are invariably also present in empirical force fields since results of quantum calculations are typically used as benchmarks for parameterizing empirical force fields. Including the variety of functionals available for describing atomic systems, we identify two other issues that can potentially affect the reliability of *ab initio* investigations on ion solvation: sampling and system size. In *ab initio* approaches that involve generation of dynamical trajectories, as in *ab initio* molecular dynamics (AIMD) simulations, the associated computational expenses prohibit effective sampling in systems containing a large number of atoms. Currently, AIMD simulations have access to a few tens of picoseconds of time-scales and most investigations are also limited to system sizes containing a few hundreds of atoms. We present below results from liquid state *ab initio* studies carried out to investigate hydration structures of the three ions, discussing alongside the effect that each of the three issues can have on the reliability of these investigations. The results on hydration numbers of the three ions have been summarized in Table 5.

Table 4
Hydration numbers of ions computed using force fields that explicitly account for atomic polarizabilities (P) and/or multi-body effects on ion–water interactions (M)

M^+	n_{M^+O}	Method	Year [Ref.]
Li^+	4	Molecular dynamics with P+M	2004 [24]
	4	Molecular dynamics with M	2004 [24]
	4	Molecular dynamics with P	2006 [82]
Na^+	6	Optimization with P+M	1985 [17]
	5, 6	Monte Carlo with P+M	2003 [83]
	6	Molecular dynamics P	2003 [12]
	7	Molecular dynamics P+M	2004 [24]
	6	Molecular dynamics with M	2004 [24]
	6	Molecular dynamics with P	2006 [82]
K^+	4+2	Optimization with P+M	1985 [17]
	7, 8	Monte Carlo P+M	2003 [83]
	7	Molecular dynamics P	2003 [12]
	7	Molecular dynamics P	2006 [82]

In 1998, Tongraar et al. [34] reported a liquid state *ab initio* investigation of the hydrated structures of Na^+ and K^+ ions. In their approach, they combined quantum mechanical and molecular mechanical (QM/MM) methods, in which the ions along with their first solvation shells were treated quantum mechanically and the remaining portion of the solution was treated using empirical force fields. This approach enabled them to circumvent both issues of sampling and system size. Their calculations produced a hydration number of 8 for K^+ ions and a hydration number of 6 for Na^+ ions. These results, however, differ substantially from results of the most recent experiments and also from any of the AIMD simulations reported in subsequent years. This is presumably because the quantum mechanical portions of their calculations were carried out at the Hartree–Fock (HF) level of theory, which neglects electron correlation effects. In contrast, electron correlation interactions are taken into account in the density functional approaches used in subsequent studies. Neglect of electron correlations result in weaker water–water interactions (see for example Ref. [35]), which leads to a smaller water dipole and thereby an underestimation of water–water repulsion. In such calculations this effect is presumably manifested as an increase in hydration numbers, although higher hydration numbers may also be an artifact of the nature of coupling between the quantum and molecular mechanical regions.

In 1999, Ramaniah et al. [36] reported the first fully *ab initio* molecular dynamics simulation investigation on K^+ ion hydration. They applied density functional theory (DFT) using the BLYP generalized gradient approximation to the electron density functional to describe the valence electrons, which accounts for electron correlations, and generated a ~2 ps long dynamics trajectory of the ion solvated by 59 water molecules. In this case interactions between all the atoms of the system were treated quantum mechanically. As part of this study, the authors also presented separate empirical force field calculations, which ascertained that their system size did not introduce artifacts in the structural analysis of K^+ ion hydration, as also established in other studies [37]. An analysis of their trajectory, however, led to a “rugged” looking radial distribution profile of water molecules around the K^+ ion, suggesting lack of equilibration. As the authors

Table 5
Hydration numbers of ions estimated using *ab initio* approaches

M^+	n_{M^+O}	Method		Year [Ref.]
		Approach	Functional	
Li^+	4	AIMD	BLYP	2001 [84]
	4	AIMD	PW91	2000 [45]
	(4)	Quasi-chemical	B3LYP	2000 [44,45]
	4	QM/MM	HF	2002 [85]
	4	QM/MM	B3LYP	2002 [85]
Na^+	6	QM/MM	HF	1998 [34]
	5	AIMD	PBE	2000 [40]
	5	AIMD	PW91	2001 [41]
	(4)	Quasi-chemical	B3LYP	2001 [41]
	4+1	AIMD	PW91	This work
K^+	8	QM/MM	HF	1998 [34]
	5.5–7.5	AIMD	BLYP	1999 [36]
	4+2	AIMD	PW91	2004 [38]
	(4)	Quasi-chemical	B3LYP	2004 [38]
	4+2	AIMD	PW91	This work

The functionals represent different implementations of gradient-corrected density functional theory except for "HF", which stands for Hartree–Fock theory. Numbers joined by a "+" refer to subpopulations found in the first peak of the ion–oxygen pair distribution functions: an innermost population of tightly coordinating water molecules and a more distant population of "loosely" coordinating water molecules, respectively. Parentheses around hydration numbers refer to the preferred number of tightly coordinating water molecules.

acknowledged, the first minimum of their radial distribution profile was not well-defined, which led them to arrive at a range of hydration numbers for K^+ ions that depended on the radial cutoff distance used for defining the inner coordination volume. The resulting hydration numbers ranged from 5.5 to 7.5 water molecules. In their analysis, the authors also found a subset of four water molecules to be most tightly bound to the K^+ ion.

In 2004, Rempe et al. [38] used a different density functional (PW91), which also accounts for electron correlations, and generated a longer time-scale (10 ps for analysis and 10 ps for equilibration) AIMD trajectory of a K^+ ion solvated by 32 water molecules. This longer simulation produced a smoother radial distribution profile of water molecules around the central K^+ ion, which by the traditional definition of coordination number resulted in a hydration number of 6, consistent with the most recent experimental data described in this issue [81]. Their radial distribution profile at the same time also presented a distinct shoulder on the far slope of the principal maximum ($r_{K-O} \sim 3$ Å), also visible in the radial distribution profile of K^+ ions obtained from an earlier AIMD simulation [39]. The analysis of their trajectory led to the finding that the first coordination shell was actually composed of two distinct subpopulations of water molecules. One subset, containing 4 water molecules, was found to occupy a region closest to the ion ($r_{K-O} < 3.4$ Å) and the other subset, with only 2 water molecules, was situated at a slightly larger distance. In other words, we see that in the case of K^+ ions, the principal occupancy in the inner coordination shell is actually 4, but at the same time there also is a statistical admixture of two other outlying oxygen atoms that blur the primary minimum of the radial distribution profile. Integration of the radial distribution profile up to this primary minimum yields a hydration number of 4, but when integrated to include the "loosely" coordinated water molecules, the hydration number is 6. Current work using a larger simulation box containing 64 water molecules produces the same results, as illustrated in Fig. 1. We indicate this characteristic inner shell occupancy of K^+ ions as 4+2 in Table 5.

In 2000, White et al. [40] applied DFT using another generalized gradient approximation to the electron density functional, the Perdew–Burke–Ernzerhof (PBE) functional, to generate a 2.5 ps long AIMD trajectory of a Na^+ ion in water. Integration of the radial distribution profile determined from this data resulted in a Na^+ hydration number of 5. In 2001, Rempe and Pratt reported [41] data from a longer (6 ps) AIMD simulation of a Na^+ ion in water using a different density functional (PW91) and arrived at the same result. The most recent neutron diffraction experiments [42], described in this issue, are consistent with this finding. The radial distribution profile reported by Rempe and Pratt [41] also indicated the presence of a tiny shoulder on the far slope of the principal maximum, but in contrast to the case of K^+ ion, the AIMD data was not analyzed any further due to the relatively smaller length of the trajectory.

To rectify the lack of detailed information about possible substructure in the first peak of the Na^+ ion radial distribution profile, we generated a longer (13 ps) trajectory of a Na^+ ion with 64 water molecules using a methodology similar to the one presented earlier [41], but with some improvements. In the new simulation, twice as many water molecules are present than in the earlier work, the 2P core electrons of Na^+ are now included explicitly, and the core–valence electron interactions are now described using the improved projector augmented-wave method. The radial distribution profile of water oxygens around Na^+ ion is illustrated in Fig. 2 and does not differ perceptibly from the earlier results. As in the case of K^+ ions, we find features at the far slope of the principle maximum rather than a smooth curve.

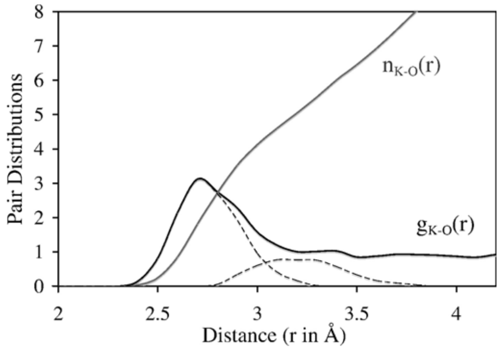

Fig. 1. Distributions of water oxygen atoms around K^+ ion estimated from a 15 ps long equilibrated AIMD trajectory using a methodology similar to the one presented earlier [38], but with some improvements. In the new simulation, twice as many water molecules are present than in the earlier work, the 3P core electrons of K^+ are now included explicitly, and the core–valence electron interactions are now described using the improved projector augmented-wave method. $g_{K-O}(r)$ indicates the density of oxygen atoms at a given radial distance from the central K^+ ion. The dashed curve closer to the central K^+ ion indicates the contribution of the nearest four water molecules to this density distribution, while the dashed curve further away from the central K^+ ion indicates the contribution from the 5th and 6th nearest water molecules. The composite radial distribution function $g_{K-O}(r)$ reflects these two subpopulations as distinct shoulders on the far slope of its principle maximum. $n_{K-O}(r)$ indicates the average number of oxygen atoms within a certain distance from the central K^+ ion.

Decomposition of the principle curve shows that the first coordination shell in this case is also composed of two subpopulations of water molecules. One subset occupies a region closest to the ion, and is composed of 4 oxygen atoms, while the other subset is situated at a slightly larger distance, and is composed of one “loosely” coordinated oxygen atom. We indicate this characteristic inner shell water occupancy of Na^+ ions as 4+1 in Table 5.

In the case of Li^+ ions, all AIMD simulations produce radial distribution profiles that have clearly defined inner coordination shells. In all studies, integration of these curves up to their respective first minima result in a Li^+ hydration number of 4. Taken together, AIMD simulations consistently predict that, in aqueous phase, all three ions prefer to strongly coordinate with exactly 4 water molecules. Additional “loosely” coordinated water molecules raise the number of water molecules within the first minimum of the pairwise radial distribution curves of Na^+ ions to 5 and K^+ ions to 6.

In the past few years, *ab initio* computations have also been utilized in connection with formal molecular association equilibrium theories developed by Pratt and coworkers to probe ion hydration numbers [38,41,43–51]. Such formal theories that describe molecular association equilibria have a very long history of development, and have been utilized on a regular basis to describe the physics of several other phenomena (see for example Refs. [52–58]). The most unique feature of the statistical mechanical formulation [38,41,43–51] constructed specifically to describe particle solvation in condensed phases is that it allows for a decoupling of the solute’s 3-dimensional interaction hyper-surface into two separate domains without consideration of the specific form of the interaction potential. Consider the following ion solvation reaction,

$$A^{+}_{\mathrm{gas}} + n[X]_M \xrightleftharpoons{K_n} [A^{+}X_n]_M \qquad (1)$$

which takes place in a certain environment M, where n independent ligands (X) from this environment react with monovalent cation A^+ to form a coordination complex A^+X_n. Here K_n represents the equilibrium constant of this reaction. Starting from the potential distribution theorem [59,60], the standard form of the chemical potential of this ion (μ_A) can be separated out into ideal and non-ideal contributions, and in a way such that the non-ideal contributions can be expressed in terms of the equilibrium constants (K_n) of these clustering reactions [43,44,58], i.e.,

$$\beta\mu_A = \ln\left[\frac{\rho_A V}{q_A}\right] - \ln\left[\left\langle \mathrm{e}^{-\beta\Delta U}\prod_j (1+f_{Aj})\right\rangle_0\right] - \ln\left[\sum_{n\geq 0} K_n(\rho_X)^n\right] \qquad (2)$$

Here ΔU represents the interaction energy between the n-fold ion ligand cluster and the rest of the system, β is the Boltzmann constant, V is the volume under consideration, ρ_A is the density of the ion, and q_A is its single molecule partition function, while ρ_X is the density of solvent molecules. f_{Aj} is the Mayer f-(cluster)-function describing the interaction between solute and solvent molecules. It is 0 when solvent j is outside the cluster, and takes a value of −1 if that solvent molecule is inside the cluster, which serves to exclude additional solvent molecules from the cluster interior.

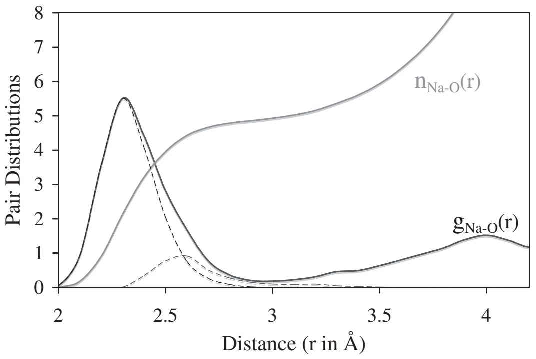

Fig. 2. Distributions of water oxygen atoms around Na^+ ion estimated from a 13 ps long equilibrated AIMD trajectory. $g_{Na-O}(r)$ indicates the density of oxygen atoms at a given radial distance from the central Na^+ ion. The dashed curve closer to the central Na^+ ion indicates the contribution of the nearest four water molecules to this density distribution, while the dashed curve further away from the central Na^+ ion indicates the contribution from the 5th nearest water molecule. The composite radial distribution function $g_{Na-O}(r)$ reflects these two subpopulations as a distinct shoulder on the far slope of its principle maximum. $n_{Na-O}(r)$ indicates the average number of oxygen atoms within a certain distance from the central Na^+ ion.

It is easy to see that such a decoupling of the net chemical potential of the ion allows for the treatment of a subsection of its spatial environment on a different footing from the rest of its environment. Furthermore, such a formulation also provides for a quasi-chemical type of physical setting in which the chemical potential can be directly probed as a function of the number and arrangement of ligands around that ion, thereby directly linking structural information to the thermodynamic properties of ion solvation. The computational implementation of this formulation has typically been carried out by treating the most significant nearest neighbor interactions of an ion quantum mechanically, and the remaining interactions using either implicit or explicit solvent models [38,41,43–47,49,51]. This circumvents critical issues that include parameterization of nearest neighbor interactions, accounting for traditional system size limitations, and determination of coupling parameters such as those present in hybrid QM/MM dynamics settings.

Such treatments applied to the phenomenon of ion hydration have yielded a tetrahedral arrangement of 4 water molecules as the preferred hydration number of all three cations considered here [38,41,44,45,47], consistent with AIMD simulation results. Interestingly, inorganic chemists also cite tetrahedral coordination for these ions in water [86]. Note that although the formal theory that has yielded these hydration numbers is exact, its implementation can still benefit from a better treatment of the thermal motions of clusters via incorporation of anharmonic modes of vibrations, and an incorporation of additional energetic terms due to dispersion and packing. Such improvements in implementation, however, are not expected to alter the predicted hydration numbers of these ions [44,47].

4. Conclusion

This review and analysis of experimental and theoretical work carried out in the last decade shows that the lack of consensus previously characterizing structural properties of hydrated ions has given way to a remarkably consistent picture of water coordination structure for the first three alkali metal ions: Li$^+$, Na$^+$, and K$^+$. Theoretical studies based on *ab initio* methods and experimental work using diffraction and spectroscopic methods converge on this surprising conclusion: exactly four (4) water molecules directly coordinate each of these ions. Additional water molecules occupy more distant but overlapping regions of space in larger ions, thus obscuring the distribution profiles of the innermost coordinating water molecules and raising the hydration numbers determined by traditional analysis of the ion–oxygen pair-distribution curves. As a result of these superimposed populations of water molecules, a unique pattern in hydration number emerges. For each unit move down the periodic table, the inner population of four coordinating water molecules is joined by additional "loosely" coordinating water molecules, one for Na$^+$ and two for K$^+$ ions, to give hydration numbers of 4, 5, and 6, respectively, for the first three alkali metal ions.

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